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Reversible quantum teleportation in an optical lattice

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Abstract
We propose a protocol, based on entanglement procedures recently suggested by Jaksch *et al*, which allows the teleportation of an unknown state of a neutral atom in an optical lattice to another atom in another site of the lattice without any irreversible detection.

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(Some figures in this article are in colour only in the electronic version)

1. Introduction

The characterization, creation, control and manipulation of quantum entanglement [1, 2] constitutes the basis of the fast-developing research field of quantum information. An entangled state of two or more [3] particles can be intuitively understood as the situation in which the state of one particle cannot be determined independently from the state of the others. Of course, the concept deserves more technical definitions and quantifications, but these are beyond the scope of this paper.

Quantum computing [4], quantum cryptography [5] and other interesting phenomena constitute excellent examples of the extraordinary potential of quantum entanglement. Among these phenomena, quantum teleportation [6] is perhaps one of the most striking. Quantum teleportation consists in the transport of a quantum state from one particle to another in a disembodied way. Following the theoretical proposal of Bennett *et al* [6], quantum teleportation has been recently demonstrated experimentally [7]. This teleportation scheme involves a Bell measurement[1], and therefore it is performed in an irreversible way due to the collapse of the wavefunction. However, it has been shown [9] that quantum teleportation can be performed in a reversible way, i.e. without any irreversible detection[2]. In order to obtain such unitary

[1] A Bell measurement consists of a joint measurement on two particles, determining whether they are in one of the four Bell states, namely, $|\phi^{\pm}\rangle = (|0\rangle|0\rangle \pm |1\rangle|1\rangle)/\sqrt{2}$, $|\psi^{\pm}\rangle = (|0\rangle|1\rangle \pm |1\rangle|0\rangle)/\sqrt{2}$. In the experiments performed up to now, only $|\psi^{+}\rangle$ and $|\psi^{-}\rangle$ can be identified, but not $|\phi^{\pm}\rangle$, whose identification is only possible if the two particles are coupled by some interaction. Concerning the problem of Bell measurements see, for example, [8].

[2] Note that in [10], reversibility just refers to Bob's step of the teleportation protocol, which as a consequence of the lack of knowledge about the teleported state is necessarily unitary.

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teleportation it is necessary to consider the detector as a quantum mechanical object, the state of which is not read out. In this paper, we show how this idea can be implemented in a particular physical situation.

Several physical systems have been proposed in which entanglement can be created and manipulated, such as cavity QED [11], photons [12] and ion-traps [13]. Recently it has been proposed that neutral atoms in an optical lattice can be entangled in a controlled way by using cold collisions between them [14]. It has been shown [15] how one can use this entanglement mechanism to create GHZ-states [3], or implement parallel quantum computing and quantum error correction [16]. We show in this paper that in the framework of this particular entanglement procedure it is possible to implement a reversible teleportation protocol.

The structure of the paper is as follows. In section 2 we review the irreversible and reversible teleportation schemes, and also present the abstract formulation of our teleportation proposal. In section 3, we briefly review the entanglement procedure of [14, 15]. In section 4, we consider explicitly the most simple case of teleportation with just three lattice sites (in appendix A we analyse the more general case of an arbitrary number of sites). We finish in section 5 with some conclusions.

2. Teleportation protocols

In this section, we review briefly the general ideas behind the teleportation schemes both with and without irreversible measurements. For a more detailed discussion we refer to [6, 9, 10]. At the end of the section we present the abstract formalism of our teleportation scheme. In this section, we follow the notation of [9].

2.1. Irreversible teleportation

We consider three two-level systems, denoted by indices 1, 2 and 3, each with basis states $\{|0\rangle, |1\rangle\}$. Initially particle 1 is in an unknown state $|\phi\rangle$, whereas particles 2 and 3 are in a maximally entangled state $|\phi^+\rangle = (|0\rangle|0\rangle + |1\rangle|1\rangle)/\sqrt{2}$. Particles 1 and 2 are together at one place, and particle 3 is at a different place. The teleportation scheme can be well understood using the decomposition [6]

$$|\phi_{\mathrm{in}}\rangle = |\phi\rangle_1|\phi^+\rangle_{23} = \frac{1}{2}\sum_{J=0}^{3}|\Psi^{(J)}\rangle_{12}R_3^{(J)\dagger}|\phi\rangle_3 \tag{1}$$

where $|\Psi^{(J)}\rangle$ is the entangled basis for two qubits,

$$\begin{aligned} |\Psi^{(J)}\rangle &= \sum_{l=0}^{1}\exp(\pi \mathrm{i} l n)|l\rangle|(l+m) \bmod 2\rangle/\sqrt{2} \\ R^{(J)} &= \sum_{k=0}^{1}\exp(\pi \mathrm{i} k n)|k\rangle\langle(k+m) \bmod 2| \end{aligned} \tag{2}$$

with

$$J = n\cdot 2 + m \qquad \text{i.e.} \quad n = J \operatorname{div} 2, m = J \bmod 2. \tag{3}$$

Explicitly this corresponds to $|\Psi^{(0)}\rangle \equiv |\phi^+\rangle, |\Psi^{(1)}\rangle \equiv |\psi^+\rangle, |\Psi^{(2)}\rangle \equiv |\phi^-\rangle, |\Psi^{(3)}\rangle \equiv |\psi^-\rangle$, the well known Bell states, and $R^{(0)} \equiv \mathbb{1}$, $R^{(1)} \equiv \sigma_1$, $R^{(2)} \equiv \sigma_3$, $R^{(3)} \equiv \mathrm{i}\sigma_2$, where σ_j are the Pauli matrices.

The teleportation scheme [6] works as follows: first a joint measurement of the state of particles 1 and 2, i.e. a Bell measurement, is performed; then, using a classical channel, the result J of the measurement is sent to the other site and, using the value J, the appropriate unitary transformation $R_3^{(J)}$ is performed on particle 3 to transform the state of particle 3 into $|\phi\rangle$. As we observe, this procedure is clearly irreversible, because a measurement of the joint state of 1 and 2 is necessary.

In the following sections a lower index 1, 2, 3, ... for a unitary transformation refers to the particle to which the transformation is applied, and a lower index a, b, c refers to the sequence of transformations.

2.2. Reversible teleportation

However, the previous scheme is not the only one which allows to teletransport an unknown state. In particular, there is a reversible way to perform such a task [9, 10]. Let us assume an auxiliary particle A (which we shall call the ancilla), which is a four-level system with basis states $\{|0\rangle, |1\rangle, |2\rangle, |3\rangle\}$. The initial state of the system (1) now becomes

$$|\phi_{\mathrm{in}}\rangle = |0\rangle_A|\phi\rangle_1|\phi^+\rangle_{23}. \tag{4}$$

The reversible teleportation scheme works as follows. First, we perform the following unitary operation on the initial state:

$$R_a = \sum_{J=0}^{3} |\Psi^{(J)}\rangle_{12} O_A^{(J)} \langle\Psi^{(J)}|_{12} \tag{5}$$

where $O_A^{(0)} \equiv \mathbb{1}$ and $O_A^{(J)} \equiv |0\rangle\langle J| + |J\rangle\langle 0| + \sum_{k\neq 0,J}^{3} |k\rangle\langle k|$, for $0 < J \leqslant 3$. It is easy to observe that the state of the system becomes

$$R_a|\phi_{\mathrm{in}}\rangle = \tfrac{1}{2}\sum_{J=0}^{3} |J\rangle_A|\Psi^{(J)}\rangle_{12} R_3^{(J)\dagger}|\phi\rangle_3. \tag{6}$$

A second step consists in transporting the particle A near particle 3, and performing a unitary transformation of the form

$$R_b = \sum_{J=0}^{3} |J\rangle_A R_3^{(J)} \langle J|_A, \tag{7}$$

i.e. a unitary transformation on 3 conditional on the value of the state of the ancilla A. After performing operation R_b the system becomes

$$R_b R_a|\phi_{\mathrm{in}}\rangle = \tfrac{1}{2}|\phi\rangle_3 \sum_{J=0}^{3} |J\rangle_A|\Psi^{(J)}\rangle_{12} \tag{8}$$

and therefore the state $|\phi\rangle$ has been transported into 3, whatever be the final value of 1, 2 and A. Eventually, if A were transported back near 1 and 2, one could perform a unitary operation to restore 1, 2 and A to their original states.

2.3. Our teleportation scheme

In this section, we present the general idea behind our reversible teleportation scheme, in order to observe the similarities and differences in comparison with the scheme presented previously. Here we consider three two-level systems, 1, 2 and 3. Note that, as shown in appendix A, the method can be generalized to an arbitrary number of particles. The lowest non-trivial number of particles is 3 and therefore is used to explain our scheme. Due to the restrictions of the physical model that we employ in section 3, all particles, including the ancillas, are in our case qubits, i.e. they have just two states $\{|0\rangle, |1\rangle\}$ (in the case of the employed ancillas we shall consider a different state $|2\rangle$ instead of $|1\rangle$). However, in order to perform the teleportation, we need two bits of information and therefore two ancillas, A_1 and A_2. We assume that the initial state of the system is of the form

$$|\phi_{\text{in}}\rangle = |0\rangle_{A_1}|0\rangle_{A_2}|\phi\rangle_1|0\rangle_2|0\rangle_3. \tag{9}$$

As one can observe, this is a difference with respect to the initial state considered in expression (1): the particles 2 and 3 are not initially entangled, and as we will see later they become entangled with each other as well as with particle 1 during our transformations.

The first step of our teleportation scheme consists in performing a unitary transformation V_a acting on the six-dimensional space of the three particles 1, 2 and 3, in such a way that the state of the system becomes

$$V_a|\phi_{\text{in}}\rangle = \sum_{J=0}^{3} |0\rangle_{A_1}|0\rangle_{A_2}|\psi^{(J)}\rangle_{12}U_3^{(J)\dagger}|\phi\rangle_3 \tag{10}$$

where the exact definitions of $|\psi^{(J)}\rangle_{12}$, $U_3^{(J)}$ and V_a will be presented in section 4. As we observe, the operation V_a entangles the particles 1, 2 and 3. The next step of the teleportation scheme is to perform a unitary operation of the form

$$V_b \equiv \sum_{J=0}^{3} |\psi^{(J)}\rangle_{12} O_{A_1A_2}^{(J)} \langle\psi^{(J)}|_{12} \tag{11}$$

where $O_{A_1A_2}^{(J)} = \sigma_{1,A_1}^{J\,\mathrm{mod}2}\sigma_{1,A_2}^{J\,\mathrm{div}2}$ (our notations are $\sigma_i^0 = \mathbb{1}$ and $\sigma_i^1 = \sigma_i$). After applying V_b the state of the system takes the form

$$V_bV_a|\phi_{\text{in}}\rangle = \sum_{J=0}^{3} |\psi'^{(J)}\rangle_{A_1A_2}|\psi^{(J)}\rangle_{12}U_3^{(J)\dagger}|\phi\rangle_3 \tag{12}$$

where the exact form of $|\psi'^{(J)}\rangle_{A_1A_2}$ is presented in section 4. This step can be called the 'reading of the states by the ancillas'. Once this is done, we perform a unitary operation of the form

$$V_c \equiv \sum_{J=0}^{3} |\psi'^{(J)}\rangle_{A_1A_2} Q_3^{(J)} \langle\psi'^{(J)}|_{A_1A_2} \tag{13}$$

where $Q_3^{(J)}U_3^{(J)\dagger} = c_J\tilde{U}_3^{(0)\dagger}$. Here the coefficients c_J can be ± 1, and are calculated in detail in section 4. Then, the state of the system becomes

$$V_cV_bV_a|\phi_{\text{in}}\rangle = U_3^{(0)\dagger}|\phi\rangle_3 \otimes \sum_{J=0}^{3} c_J|\psi'^{(J)}\rangle_{A_1A_2}|\psi^{(J)}\rangle_{12} \tag{14}$$

where we have moved the state of 3 out of the sum to stress that it is now independent of the state of 1, 2 and the ancillas. As a final step, we just need to perform the unitary operation $U_3^{(0)}$ to conclude the teleportation. The ancillas and 1 and 2 remain in an entangled state. Finally, but this is not necessary for the teleportation, if we perform again V_b the ancillas are brought back to their original state $|0\rangle_{A_1}|0\rangle_{A_2}$.

3. Quantum entanglement of atoms in optical lattices

In this section, we present the physical system in which we shall implement our reversible teleportation scheme, briefly reviewing [14, 15]. Let us consider a collection of bosonic neutral atoms occupying the sites of an optical lattice. In order to perform the necessary quantum logical operations, one has to be able to fill the lattice wells with exactly one particle each. This can be achieved—at present only theoretically—by loading the lattice from a Bose–Einstein condensate (BEC), and inducing at sufficiently low temperatures a phase transition from the superfluid BEC phase into a Mott insulator phase, by increasing the ratio between the interaction energy inside each well and the tunnelling rate between the wells, as predicted by the Bose–Hubbard model [17].

One can perform in the system one-atom operations by shining a laser on a desired atom. However, it is not realistic to assume that only one atom is then affected, because the atoms are typically separated by a $\lambda/2$ distance, where λ is the wavelength of the laser which creates the lattice. Fortunately, our teleportation scheme does not employ one-atom unitary operations specifically for one site, but operations which can, in principle, be applied simultaneously to all the atoms of the lattice[3]. In addition to these single-atom operations, two basic two-atom operations can be perfomed within this physical scheme, based on cold collisions between the atoms.

3.1. Shift operation

The two internal states of the atoms carrying the quantum information are called $\{|0\rangle, |1\rangle\}$. As shown in detail in [15], by properly arranging the detuning and polarization of the lasers which form the lattice, it can be achieved that each of the internal atomic levels can have a different potential. In particular, by changing the dephasing between the circularly polarized waves $\sigma^{\pm}$ which form the lattice, the potentials for the different internal levels move in opposite directions. Let us suppose that the lattice of $|0\rangle$ moves to the right, while the lattice of $|1\rangle$ moves to the left. Two neighbouring atoms in the lattice occupy sites j and $j+1$ (our numbering is from left to right). It is clear that, using a lattice displacement, the neighbouring atoms can only undergo a collision if the atom at j is in $|0\rangle$ and the one at $j+1$ is in $|1\rangle$. Any other situation prevents the atoms from approaching each other. When the two particles are put in contact, they can interact via s-wave scattering. As a result, a collisional phase appears, which can be controlled by basically changing the interaction time. In particular, we shall choose this collisional phase to be π. After the desired time the lattice is brought back to its original position[4]. We shall call this operation a *shift operation*, following the notation of [15], and it can be summarized as follows:

$$\begin{aligned}
|0\rangle_j|0\rangle_{j+1} &\to |0\rangle_j|0\rangle_{j+1} \\
|0\rangle_j|1\rangle_{j+1} &\to -|0\rangle_j|1\rangle_{j+1} \\
|1\rangle_j|0\rangle_{j+1} &\to |1\rangle_j|0\rangle_{j+1} \\
|1\rangle_j|1\rangle_{j+1} &\to |1\rangle_j|1\rangle_{j+1}.
\end{aligned} \tag{15}$$

[3] The only exception is the ancilla, which is assumed to be placed sufficiently far away from the other atoms of the lattice, and therefore individual unitary operations can be performed on this particular atom.

[4] In principle, kinetic phases appear as well, due to the movement of the atoms, but these are trivial one-particle phases, which can always be incorporated into the definition of states $|0\rangle$ and $|1\rangle$.

3.2. Sweep operation

Let us assume that the atoms have a third atomic level $|2\rangle$, which can be displaced like the levels $|0\rangle$ and $|1\rangle$ by using the corresponding transport lattice. The operation is basically like the previous one, but now only those atoms in level $|2\rangle$ are going to participate. In particular, we are going to consider that just the ancilla is excited into the level $|2\rangle$. The interaction of the ancilla with an atom in the site j can be designed (following the same arguments as above) in such a way that[5]

$$\begin{aligned} &|2\rangle_A|0\rangle_j \rightarrow |2\rangle_A|0\rangle_j \\ &|2\rangle_A|1\rangle_j \rightarrow -|2\rangle_A|1\rangle_j. \end{aligned} \tag{16}$$

Following [15], we shall call this operation a *sweep operation*. By varying the speed with which the lattice of the state $|2\rangle$ is moved during the sweep operation, it is possible to act on a particular site of the lattice, even when the ancilla crosses through other sites in the lattice, in particular because the collisional time can be designed in such a way that for the undesired sites the collisional phase is a multiple of 2π.[6]

4. Reversible teleportation protocol: three sites only

Let us now show explicitly the transformations for our scheme of reversible teleportation. In this section we explain the most simple case, in which we have three sites, each occupied by one atom. In appendix A we analyse the more general case of an arbitrary number of lattice sites. We shall call these atoms (from left to right in the lattice) 1, 2 and 3. We shall consider another two atoms A_1 and A_2, which will act as ancillas, and which are initially placed sufficiently apart (to the left) from the sites 1, 2 and 3; this requirement is necessary to avoid the possibility that the unitary operations applied on the site 1, 2 and 3 could affect the ancillas, and vice versa. We shall also consider that both the ancillas are separated by a distance larger than the dimensions of the three sites 1, 2 and 3; this requirement is necessary to avoid the possibility that during the operation of one ancilla on the sites, the other sites could be affected by the other ancilla. We shall discuss in section 5 the case in which only one ancilla is present. We begin with the initial state of the system

$$|\phi_{\rm in}\rangle = |0\rangle_{A_1}|0\rangle_{A_2}|\phi\rangle_1|0\rangle_2|0\rangle_3 \tag{17}$$

where $|\phi\rangle = a|0\rangle + b|1\rangle$. Our objective is to transport this state from particle 1 to particle 3, in a reversible way, using the operations of section 3. Our teleportation scheme consists of three general steps, described in the following three subsections.

4.1. Creation of the entanglement between 1, 2 and 3

As a first step, we perform a Hadamard transform (H)

$$\begin{aligned} &|0\rangle \rightarrow \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \\ &|1\rangle \rightarrow \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) \end{aligned} \tag{18}$$

[5] In principle, $|2\rangle_A|0\rangle_j \rightarrow e^{i\phi_0}|2\rangle_A|0\rangle_j$, whereas $|2\rangle_A|0\rangle_j \rightarrow e^{i(\phi_0+\phi)}|2\rangle_A|1\rangle_j$, but as the phase ϕ_0 appears anyway, it can be reabsorbed in the definition of $|2\rangle_A$, as is also done with the kinetic phases. By setting $\phi = \pi$, we retrieve expression (16).

[6] As pointed out in [15], another, perhaps more elegant, way to solve this problem is to consider a three-dimensional lattice, in which the ancilla can be vertically displaced upwards, moved, and displaced downwards to the desired site of the lattice.

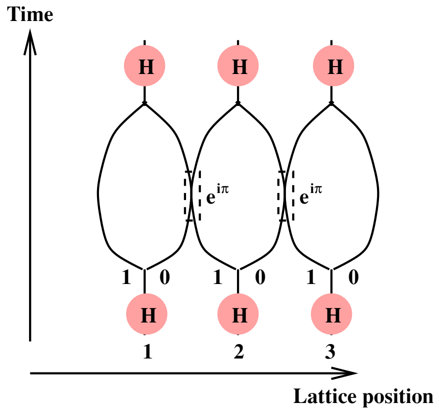

Figure 1. Creation of the entanglement between sites 1, 2 and 3. We have depicted the shift operation as a multiparticle interferometer following [15]. Here H denotes a Hadamard transform.

in each one of the sites 1, 2 and 3. Then, we perform a shift operation (of the lattices of $|0\rangle$ and $|1\rangle$), and after this we perform another Hadamard transformation to all the sites. As a result of these three operations, the state of the system becomes

$$V_a|\phi_{\rm in}\rangle = \tfrac{1}{2}|0\rangle_{A_1}|0\rangle_{A_2}\sum_{J=0}^{3}|\psi^{(J)}\rangle_{12}U_3^{(J)\dagger}|\phi\rangle_3 \tag{19}$$

where $|\psi^{(J)}\rangle_{12} = |J\,{\rm div}\,2\rangle_1|J\,{\rm mod}\,2\rangle_2$, and $U^{(0)} = -{\rm i}\sigma_2$, $U^{(1)} = \sigma_1$, $U^{(2)} = -\sigma_3$, $U^{(3)} = \mathbb{1}$. As we observe in figure 1, the states of the three sites now become entangled.

4.2. Reading of 1 and 2 using the ancillas

First, a Hadamard transform is performed in each of the ancillas (between levels $|0\rangle$ and $|2\rangle$),

$$H_A V_a|\phi_{\rm in}\rangle = \tfrac{1}{4}(|0\rangle_{A_1} + |2\rangle_{A_1})(|0\rangle_{A_2} + |2\rangle_{A_2}) \otimes \sum_{J=0}^{3}|\psi^{(J)}\rangle_{12}U_3^{(J)\dagger}|\phi\rangle_3. \tag{20}$$

Then, we perform a sweep to the right of the lattice of $|2\rangle$ until the ancilla A_1 is placed in site 2, and after this, sweeping again to the right we place the ancilla A_2 in site 1. In both cases, the interaction times are properly designed to obtain a sweep operation as described in (16). Finally, we displace the lattice of $|2\rangle$ back to its original position (see figure 2). The state of the system after this operation $\tilde{V}_b$ becomes

$$\begin{aligned}\tilde{V}_b H_A V_a|\phi_{\rm in}\rangle = \tfrac{1}{4}\sum_{J=0}^{3}&\left(|0\rangle_{A_1} + (-1)^{J{\rm mod}2}|2\rangle_{A_1}\right)\\ &\times\left(|0\rangle_{A_2} + (-1)^{J{\rm div}2}|2\rangle_{A_2}\right)\otimes|\psi^{(J)}\rangle_{12}U_3^{(J)\dagger}|\phi\rangle_3.\end{aligned} \tag{21}$$

Performing a new Hadamard transform in both the ancillas, the state of the system becomes

$$V_b V_a|\phi_{\rm in}\rangle = \tfrac{1}{2}\sum_{J=0}^{3}|\psi'^{(J)}\rangle_{A_1A_2}|\psi^{(J)}\rangle_{12}U_3^{(J)\dagger}|\phi\rangle_3 \tag{22}$$

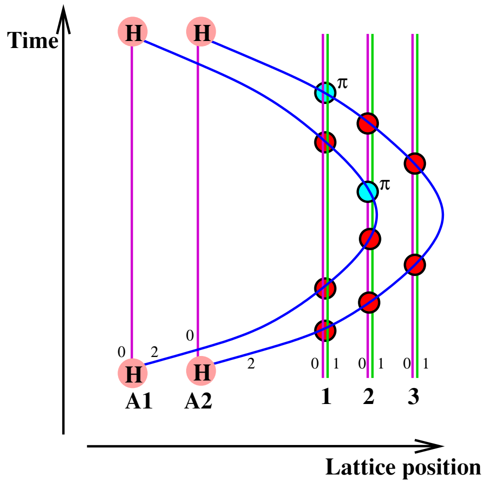

Figure 2. Reading the states of the sites 1 and 2 using the ancillas A_1 and A_2. H denotes a Hadamard transform, and the circles denote sweep operations. The sweep operations which introduce a phase π as in (16) are indicated. The rest are assumed to lead to a zero phase.

where $|\psi'^{(J)}\rangle_{A_1A_2} = |J \bmod 2\rangle_{A_1}|J \operatorname{div} 2\rangle_{A_2}$ and $V_b = H_A\tilde{V}_bH_A$. Therefore, we have copied the state of 1 (2) into A_2 (A_1). Note that the joint operation V_b is equivalent to a sequence of two CNOT gates: (i) with 2 as control qubit and A_1 as target; (ii) with 1 as control qubit and A_2 as target.

4.3. Teleportation

Now, we are going to use the values of the ancillas to teleport the state $|\phi\rangle$ into the site 3. First of all, we perform a Hadamard transform in the site 3 (in principle this operation can be performed simultaneously in the other two sites, but since the other sites remain untouched during this step, we just consider for simplicity that only the site 3 is affected by this operation). Then, the lattice of $|2\rangle$ is swept to the right until $|2\rangle_{A_2}$ is in contact with the site 3 and interacts following the rule of (16). Let us call this operation $\tilde{V}_c$. After this, we perform again a Hadamard transform in the site 3 (see figure 3). One calculates that the effect of these steps is to change the state of the system into

$$\frac{1}{2}\left(|\psi'^{(0)}\rangle_{A_1A_2}|\psi^{(0)}\rangle_{12} - |\psi'^{(2)}\rangle_{A_1A_2}|\psi^{(2)}\rangle_{12}\right)U_3^{(0)\dagger}|\phi\rangle_3 \\ + \frac{1}{2}\left(|\psi'^{(1)}\rangle_{A_1A_2}|\psi^{(1)}\rangle_{12} + |\psi'^{(3)}\rangle_{A_1A_2}|\psi^{(3)}\rangle_{12}\right)U_3^{(1)\dagger}|\phi\rangle_3. \tag{23}$$

After this, we move further to the right of the lattice of $|2\rangle$ until $|2\rangle_{A_1}$ enters in contact with the site 3, interacts with it following the rule of (16), and then we sweep back the lattice of $|2\rangle$ to its original position. Let us call this operation $\tilde{V}_c'$. After the joint operation $V_c = \tilde{V}_c' H_A \tilde{V}_c H_A$, the state of the system takes the form

$$V_cV_bV_a|\phi_{\text{in}}\rangle = \frac{1}{2}\left(|\psi'^{(0)}\rangle_{A_1A_2}|\psi^{(0)}\rangle_{12} - |\psi'^{(1)}\rangle_{A_1A_2}|\psi^{(1)}\rangle_{12}\right. \\ \left. - |\psi'^{(2)}\rangle_{A_1A_2}|\psi^{(2)}\rangle_{12} - |\psi'^{(3)}\rangle_{A_1A_2}|\psi^{(3)}\rangle_{12}\right)U_3^{(0)\dagger}|\phi\rangle_3. \tag{24}$$

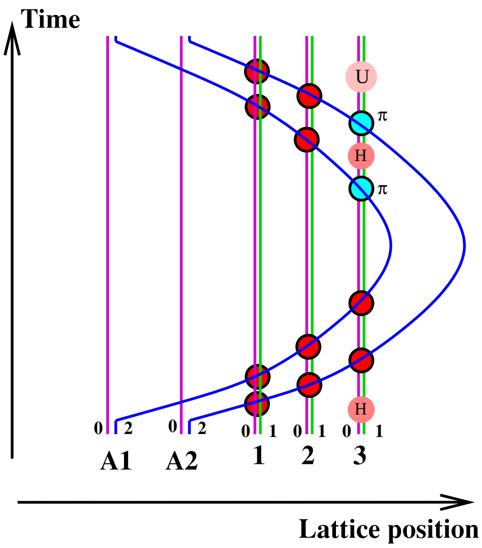

Figure 3. Using the values deposited in the ancillas during the reading process, the teleportation is finalized using unitary one-atom operations in site 3, and one sweep of the lattice of the state $|2\rangle$. In the graphic, H denotes the Hadamard transform, $U \equiv U^{(0)}$, and the circles denote sweep operations. The sweep operations which introduce a phase π as in (16) are indicated. The rest are assumed to lead to a zero phase.

Then, we just need to perform the unitary operation $U_3^{(0)}$ to complete the teleportation. Eventually, if we perform again the operations of reading the ancilla, it is possible to bring back the ancillas to their initial value $|0\rangle_{A_1}|0\rangle_{A_2}$, but this is not necessary for the teleportation.

5. Conclusions

In this paper we have presented a teleportation scheme *which allows to teleport* an atomic state of a two-level atom confined in some site of an optical lattice to another two-level atom in a distant site of the lattice, in a reversible way. In order to achieve this, we have used entanglement procedures recently developed in [14, 15]. We have shown the similarities and differences of our teleportation scheme in comparison with other reversible teleportation schemes. One difference is that we begin the teleportation process with a disentangled system, and perform a shift operation which entangles all the sites of the lattice; in particular, the particle which possesses initially the state we want to teleport, and the one in which we want to put the state at the end of the process, are entangled by this operation. We have shown that by using two other atoms (ancillas), we can teleport the desired state. This is achieved using basically unitary one-atom operations (remember that it is not necessary to address just a specific lattice position in these operations) and sweep operations; in particular, only two sweeps of the lattice of a third atomic level $|2\rangle$ are necessary. The process is fully realized via unitary transformations, and no measurement is performed; therefore, the process is completely reversible.

Let us make some remarks concerning other aspects of the suggested scheme. The state $|\phi\rangle$ can be initialized in the site 1 following a procedure similar to that which can be employed in the reading step of the teleportation scheme: (a) we consider an ancilla A sufficiently separated from the rest of the atoms, and illuminate with a laser in such a way that a state $|\phi\rangle_A = a|0\rangle_A + b|2\rangle_A$ is created; (b) then, we perform a Hadamard transform in the site 1, and perform a sweep operation between A and 1; (c) we perform a Hadamard transform in the ancilla, and also a second sweep operation between A and 1. The result is that the ancilla becomes $|0\rangle_A$, while the site 1 acquires the state $|\phi\rangle_1$, as desired. In a similar way, we can put the final state of the site N into a sufficiently isolated ancilla and perform a fluorescence experiment by shining with a laser. This allows us to read the final state, showing that the teleportation has been actually produced.

We want to emphasize that the purpose of this paper is to explain a possible implementation of a reversible teleportation scheme in an optical lattice, and *not* to show the most simple way to transport the state $|\phi\rangle$ from one lattice site to another. This could have been achieved more easily by swapping the state of the first site with the ancilla as described earlier and then the state of the ancilla with the second site. The difference between our scheme and simple swapping is that in our case the information is never transported to and localized in the ancilla, but spread over the total state and then localized in the second site, therefore exhibiting a non-trivial quantum operation.

Finally, we have used two ancillas, due to the fact that the ancillas are two-level atoms, and therefore cannot store four different values as required in the teleportation scheme. The teleportation can also be performed with just one ancilla, but the scheme becomes more complicated. Basically what is needed is a three-step process: (i) first the ancilla reads the even sites of the lattice (as shown in appendix A), and after this it is brought to the last lattice site N; after this step the possible states of the site N become two instead of four; (ii) the reading process is repeated, bringing the ancilla to its original state $|0\rangle$; (iii) then, the ancilla reads the odd states as in appendix A, and it is brought to N; after this step the possible states of the site N are reduced to just one, and therefore one needs only to perform a known unitary transformation to conclude the teleportation.

We hope that the presented teleportation protocol will motivate further efforts towards the realization of simple quantum networks in optical lattices.

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Appendix A. Generalization to an arbitrary number of sites

In this section we shall show how our teleportation scheme can be extended to the case in which we have N sites, instead of three as in section 4. We shall consider that N is an even number, i.e. $N = 2m$, but similar procedures can be designed for odd N (as already shown in the case of $N = 3$ in section 4). Therefore, our physical system is now composed of N two-level atoms each in one site of the lattice, and two ancillas. We consider that the initial state of the system is of the form

$$|\phi_{\rm in}\rangle = |0\rangle_{A_1}|0\rangle_{A_2}|\phi\rangle_1 \bigotimes_{j=2}^{N} |0\rangle_j. \tag{A1}$$

Appendix A.1. Creation of the entanglement between $1, 2, \ldots, N$

As in the case of three sites, we perform as a first step a Hadamard transform in each one of the sites $1, \ldots, N$. Then, we perform a shift operation (of the lattices of $|0\rangle$ and $|1\rangle$), and after this we perform another Hadamard transformation on all the sites. In appendix B, we demonstrate that after applying the previous three operations the state of the system becomes (remember that $m = N/2$)

$$V_a|\phi_{\rm in}\rangle = \frac{1}{2^m}|0\rangle_{A_1}|0\rangle_{A_2}\sum_{J=0}^{2^N-1} c_J|\psi^{(J)}\rangle_{1,\ldots,N-1}U_N^{(J)\dagger}|\phi\rangle_N \tag{A2}$$

where

$$|\psi^{(J)}\rangle_{1,\ldots,N-1} = \bigotimes_{k=1}^{N-1}|a_k^{(J)}\rangle_k \tag{A3}$$

with $J = \sum_{k=1}^{N-1} a_k^{(J)}2^{N-1-k}$, and c_J can be ± 1. We define

$$S_e^{(J)} = \sum_{k=1}^{m-1} a_{2k}^{(J)} \tag{A4}$$

$$S_o^{(J)} = \sum_{k=1}^{m} a_{2k-1}^{(J)} \tag{A5}$$

which count the number of $|1\rangle$s in the even sites (except N) and in the odd sites, respectively. We show in appendix B that the following holds if m is an even number:

- if $S_e^{(J)} \bmod 2 = 0$, $S_o^{(J)} \bmod 2 = 0$, then $U_N^{(J)} = W^{(0)} \equiv 1 + \mathrm{i}\sigma_2$.
- if $S_e^{(J)} \bmod 2 = 1$, $S_o^{(J)} \bmod 2 = 0$, then $U_N^{(J)} = W^{(1)} \equiv 1 - \mathrm{i}\sigma_2$.
- if $S_e^{(J)} \bmod 2 = 0$, $S_o^{(J)} \bmod 2 = 1$, then $U_N^{(J)} = W^{(2)} \equiv \sigma_3 + \sigma_1$.
- if $S_e^{(J)} \bmod 2 = 1$, $S_o^{(J)} \bmod 2 = 1$, then $U_N^{(J)} = W^{(3)} \equiv \sigma_3 - \sigma_1$.

If m is odd the same is valid, but one has to interchange $S_{e,o}^{(J)} \bmod 2 = 0 \leftrightarrow S_{e,o}^{(J)} \bmod 2 = 1$.

Appendix A.2. Reading of $1, \ldots, N-1$ *using the ancillas*

As in the case of three sites, we first perform a Hadamard transform in each of the ancillas (between levels $|0\rangle$ and $|2\rangle$),

$$H_A V_a|\phi_{\rm in}\rangle = \frac{1}{2^{m+1}}(|0\rangle_{A_1} + |2\rangle_{A_1})(|0\rangle_{A_2} + |2\rangle_{A_2}) \otimes \sum_{J=0}^{2^N-1} c_J|\psi^{(J)}\rangle_{1,\ldots,N-1}U_N^{(J)\dagger}|\phi\rangle_N. \tag{A6}$$

Then, we perform a sweep to the right of the lattice of $|2\rangle$ until placing the ancilla $|2\rangle_{A_1}$ in site $N-1$. Then, we displace $|2\rangle_{A_1}$ to the left, in such a way that a sweep operation (16) is performed every two sites beginning with $N-1$, i.e. in the sites $N-1, N-3, \ldots, 3, 1$. After this we sweep back to the right of the ancilla state $|2\rangle_{A_2}$ in such a way that a sweep operation (16) is performed every two sites beginning with $N-2$, i.e. in the sites $N-2, N-4, \ldots, 4, 2$. Finally, we displace the lattice of $|2\rangle$ back to its original position. The state of the system becomes

$$\begin{aligned}\tilde{V}_b H_A V_a|\phi_{\rm in}\rangle = \frac{1}{2^{m+1}}\sum_{J=0}^{2^N-1} c_J\left(|0\rangle_{A_1} + (-1)^{S_e^{(J)} \bmod 2}|2\rangle_{A_1}\right)\\ \times\left(|0\rangle_{A_2} + (-1)^{S_o^{(J)} \bmod 2}|2\rangle_{A_2}\right) \otimes |\psi^{(J)}\rangle_{1,\ldots,N-1}U_N^{(J)\dagger}|\phi\rangle_N.\end{aligned} \tag{A7}$$

Performing a new Hadamard transform in both the ancillas, the state of the system becomes

$$V_b V_a|\phi_{\text{in}}\rangle = \frac{1}{2^m}\sum_{J=0}^{2^N-1} c_J|S_e^{(J)} \bmod 2\rangle_{A_1}|S_o^{(J)} \bmod 2\rangle_{A_2}|\psi^{(J)}\rangle_{1\ldots N-1} U_N^{(J)\dagger}|\phi\rangle_N \tag{A8}$$

with $V_b = H_A\tilde{V}_b H_A$.

Appendix A.3. Teleportation

As in section 4, we are going to use the values of the ancillas to teleport the state $|\phi\rangle$ into the site N. First, we perform a Hadamard transform in the site N. Then, the lattice of $|2\rangle$ is swept to the right until $|2\rangle_{A_2}$ is in contact with site N and interacts following the rule of (16). Let us call this operation $\tilde{V}_c$. After that we perform again a Hadamard transform in the site N. The effect of these steps is to change the state of the system into

$$\frac{1}{2^m}\sum_{J=0, S_o^{(J)} \bmod 2=0}^{2^N-1} c'_J|S_e^{(J)} \bmod 2\rangle_{A_1}|0\rangle_{A_2}\otimes|\psi^{(J)}\rangle_{1\ldots N-1}W_N^{(0)\dagger}|\phi\rangle_N$$
$$+\frac{1}{2^m}\sum_{J=0, S_o^{(J)} \bmod 2=1}^{2^N-1} c'_J|S_e^{(J)} \bmod 2\rangle_{A_1}|1\rangle_{A_2}|\psi^{(J)}\rangle_{1\ldots N-1}W_N^{(1)\dagger}|\phi\rangle_N, \tag{A9}$$

where c'_J can be ±1. After this, we move further to the right of the lattice of $|2\rangle$ until $|2\rangle_{A_1}$ is in contact with site N, interacts with it following the rule of (16), and we sweep back the lattice of $|2\rangle$ to its original position. Let us call this operation $\tilde{V}'_c$. After the joint operation $V_c = \tilde{V}'_c H_A\tilde{V}_c H_A$, the state of the system takes the form

$$V_c V_b V_a|\phi_{\text{in}}\rangle = \frac{1}{2^m}\left[\sum_{J=0}^{2^N-1} c''_J|S_e^{(J)} \bmod 2\rangle_{A_1}|S_o^{(J)} \bmod 2\rangle_{A_2}|\psi^{(J)}\rangle_{1\ldots N-1}\right]\otimes W_N^{(0)\dagger}|\phi\rangle_N. \tag{A10}$$

where again c''_J can be ±1. Then, we just need to perform the unitary operation $W_N^{(0)}$ to complete the teleportation. As in section 4, if we perform again the operations of reading the ancilla, it is possible to bring back the ancillas to their initial value $|0\rangle_{A_1}|0\rangle_{A_2}$, but as in the previous case this is not necessary for the teleportation.

Appendix B. Proof of equation (A2)

In this appendix we prove that after applying in the initial state (A1) a Hadamard transform in all the N sites (here we denote this operation as $H^{\oplus N}$), performing a shift operation (in the following we call it L), and applying again $H^{\oplus N}$, the state of the system becomes that of expression (A2). We prove this using induction arguments. Let us call $\mathcal{Q}_N = H^{\oplus(N)}LH^{\oplus(N)}$. It is easy to observe that for the case of four sites, expression (A2) is fulfilled. Let us assume that for the case of $N = 2m$ sites, (A2) is fulfilled, i.e.

$$|\Phi\rangle_{1,\ldots,N} = \frac{1}{2^m}\sum_{J=0}^{2^N-1} c_J|\psi^{(J)}\rangle_{1\ldots N-1}U_N^{(J)\dagger}|\phi\rangle_N \tag{B1}$$

with $c_J = \pm1$. Now, we will add two more sites $(N+1)$ and $(N+2)$ to the right of the site N. The effect of $\mathcal{Q}_{N+2}$ on the initial state for the $N+2$ sites can be easily calculated from the state $|\Phi\rangle_{1,\ldots,N}$ using the unitary character of the operations; then

$$
\begin{aligned}
|\Phi\rangle_{1,\ldots,N+2} &= \mathcal{Q}_{N+2}\left[\mathcal{Q}_N^{-1}|\Phi\rangle_{1,\ldots,N}\otimes|0\rangle_{N+1}|0\rangle_{N+2}\right] \\
&= \frac{1}{2^{m+1}}\sum_{J=0}^{2^N-1} c_J|\psi^{(J)}\rangle_{1\ldots N-1}\otimes\sum_{k=0}^{3} b_k|k\,\mathrm{div}\,2\rangle_N|k\,\mathrm{mod}\,2\rangle_{N+1}O^{(k)}U_N^{(J)\dagger}|\phi\rangle_{N+2} \\
&= \frac{1}{2^{m+1}}\sum_{J=0}^{2^{N+2}-1} c'_J|\psi'^{(J)}\rangle_{1\ldots N+1}U_{N+2}'^{(J)\dagger}|\phi\rangle_N
\end{aligned}
\tag{B2}
$$

where $O^{(0)}\equiv -\mathrm{i}\sigma_2$, $O^{(1)}\equiv\sigma_1$, $O^{(2)}\equiv\sigma_3$, $O^{(4)}\equiv \mathbb{1}$, and b_k and c'_J can be ± 1. Let us assume that m is even, and therefore $U_N^{(J)\dagger}$satisfies the requirements of appendix A. It is possible to obtain, after some calculations, that the new unitary operators $U_{N+2}'^{(J)\dagger}$ satisfy the same requirements but interchange $S_{e,o}^{(J)}\,\mathrm{mod}\,2 = 0 \leftrightarrow S_{e,o}^{(J)}\,\mathrm{mod}\,2 = 1$. We note that the same applies if m is odd. Therefore, we have proved that if for N sites (A2) is satisfied, this also holds for $N+2$ sites. Since for $N=4$ the statement is true, the proof is completed by induction.

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